Adding Electron Physics to BATSRUS for CRASH

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Summary

The CRASH experiment consists of multiple materials (Xe, Be, and plastic) through which a fast shock of the order of 150 km/s will be launched. The materials are at sufficiently high temperatures that they will ionize and the free electrons will emit and absorb x-rays. The emission of the radiation near the shock, and the resulting non-local transport of energy to remote parts of the system, will affect the evolution of the shock.

A single fluid description is used to describe the CRASH experiment, covering all of the atomic and ionic species, and the electrons in the plasma. We solve the hydrodynamic model, driven by radiation energy and momentum deposition and closed by the equation of state. The gray radiation diffusion model is used, with opacity data that depends on the transport of energy to remote parts of the system, will affect the evolution of the shock.

The hydro state variables. The equations for (near) conservation of mass, momentum, ion hydrodynamic variables and O(1) of state. The gray radiation diffusion model is used, with opacity data that depends on the model, driven by radiation energy and momentum deposition and closed by the equation of state. The CRASH experiment consists of multiple materials (Be and Xe) with different radiation opacities. We use 1200 cells and solve with the HLLE numerical scheme.

Godunov solver

For the second-order Godunov numerical scheme, we apply a limited reconstruction procedure together with an exact Riemann solver for constant polytropic index, \( \gamma = 5/3 \). We add up all pressures before applying the Godunov scheme to have a upper bound for the speed of sound:

\[
c = \sqrt{\frac{\gamma p}{\rho}}
\]

To correct for the polytropic index of isotropic radiation, \( \gamma = 4/3 \), we use the fact that 4/3 is the average of 1 and 5/3 so we average the isothermal and adiabatic expansion of the radiation pressure to get the true expansion.

To correct for the spatially varying \( \gamma_e \), we use the method of artificial relaxation (see below).

Artificial relaxation for electron energy

To account for the spatially varying polytropic index of the electrons, \( \gamma_e \), due to ionization, excitation, and Coulomb interactions, we use the method of artificial relaxation to combine constant gamma hydro solvers with an accurate EOS solver:

1. At the beginning of the time step define the extra internal energy \( \Delta \varepsilon_e \), as the difference between the true electron internal energy and the translational electron energy:

\[
\Delta \varepsilon_e = \varepsilon_e - \frac{T_e}{\gamma_e - 1}.
\]

2. Assume the electron energy to be the sum of two conservation laws (\( \Delta \varepsilon_e \geq 0 \), i.e. \( \gamma_e \) for an overestimate of the speed of sound or underestimate for the time step):

\[
\frac{\partial}{\partial t} \left[ \frac{p_e (E_e - \varepsilon_e)}{\gamma_e - 1} \right] + \nabla \cdot \left[ \frac{p_e}{\gamma_e - 1} E_e \right] = 0,
\]

\[
\frac{\partial}{\partial t} \left[ \frac{1}{\gamma_e - 1} \varepsilon_e \right] + \nabla \cdot \left[ \frac{E_e}{\gamma_e - 1} \varepsilon_e \right] = 0.
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3. Advance the hydro equation through one time step. Subsequently recover the true electron energy: \( \varepsilon_e = \frac{p_e}{\gamma_e - 1} + \Delta \varepsilon_e \).

4. Recover the electron pressure from the updated electron energy and mass density by means of the electron EOS: \( p_e = \rho \varepsilon_e (\gamma_e - 1) \).

Reinieke Meyer-ter Vehn test

The Reinieke Meyer-ter Vehn test considers a initially strong and concentrated explosion in an ambient gas for which \( T_0 = T_i \), and no radiation. The heat conduction dominates the fluid flow, leading to a sizeable thermal front leading the shock front. This test uses a nonlinear heat conduction model:

\[
C_e = T_e^{\gamma - 3} / \rho^2
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The spherical blast wave is calculated in the rz-geometry and compared with the known self-similar solutions.

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Full system test

Three different full system 1D CRASH test results are shown below, which from left to right are respectively:

1. Two-temperature model with \( T_e = T_i \) (no electron energy) and radiation \( T_r \)

2. Two-temperature model with \( T_e \neq T_i \) (electron heat conduction and \( T_r \) relaxation), but no radiation

3. Three-temperature model: \( T_e, T_i, \) and \( T_r \)

The first part of the simulation is performed with the Hyades code using multi-group radiation and \( T_e \neq T_i \) and runs till 1.1 ns. After that the CRASH code takes over the simulations for 12 ns. The multi-material (Be and Xe) is represented by level sets (no mixtures) and uses non-ideal EOS. The radiation model is gray diffusion with tabular opacities. We use 1200 cells and solve with the HLLE numerical scheme.

200x200 resolution

400x400 resolution

All three simulations show the shock-relaxation structure. For case (1), the relaxation is due energy exchange between matter and radiation (Planck opacity). In case (2), the relaxation is caused by the electron-ion interaction rate (4x).

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