Level Set Methods for Multi-Material Radiation Hydrodynamics

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**The Level Set Method**

Level sets are an efficient method for tracking material interfaces in multi-material flows.

Two compressible ideal flows are separated by an interface, which is embedded as a level-set of a function \( \psi(x, t) \). At \( t = 0 \):
- \( \psi(x, 0) > 0 \) in fluid A
- \( \psi(x, 0) < 0 \) in fluid B
- \( \psi(x, t) = 0 \) at interface

The interface propagates with fluid velocity:

\[
\psi_t + u \cdot \nabla \psi = 0
\]

For \( t > 0 \):
- \( \psi(x, t) > 0 \) \( \Rightarrow \) fluid A
- \( \psi(x, t) < 0 \) \( \Rightarrow \) fluid B
- \( \psi(x, t) = 0 \) locates interface

The EOS for two compressible ideal fluids is:

\[
p = (\gamma(\psi) - 1)(E - \frac{1}{2} \rho u^2)
\]

\[
\gamma = \gamma(\psi) = \begin{cases} \gamma_A, & \psi < 0 \\ \gamma_B, & \psi > 0 \end{cases}
\]

**Spurious Pressure Fluctuations**

Pressure is constant across contact surfaces. Finite volume methods ‘smear’ interfaces. Conservative schemes do not preserve constant pressure across diffused material fronts.

Pressure oscillations:
- Occur with state-of-the-art methods
- Occur in first order schemes
- Are not improved by mesh refinement
- Are not improved by higher-order schemes

**Importance to CRASH**

CRASH solves multi-dimensional hydro problem with 3 or more fluid components, each coupled to radiation transport. Level sets are utilized.

**Method:**
- Determine interface location using level set
- Solve conservative system everywhere
- Across interface, replace energy eqn with pressure eqn
- Compute total energy using EOS

**NOTES:**
- Level set eqn may be cast in conservation form:
  \[
  (\rho u)_t + \nabla \cdot (\rho u u) = 0.
  \]

**1-D Numerical Experiment**

A 1-D rad-hydro model with gray (diffusion) transport plus a level set:

\[
\begin{pmatrix}
\rho \\
\rho u \\
E \rho \\
E \rho u
\end{pmatrix}_t + \begin{pmatrix}
\rho u \\rho u^2 + p + \frac{1}{3} E_{rad} \\
u(E + p) \\
\rho u \rho u
\end{pmatrix}_x = \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

\[
(E_{rad})_t + (u E_{rad})_x + \frac{1}{3} E_{rad} = S + D E_{rad}
\]

- Ideal gas EOS for each species
- Radiation pressure: \( p_{rad} = \frac{E_{rad}}{3} \)
- Coupling source: \( S = R \left( \frac{3}{4} \right) - AE_{rad} \)
- Parameters \( R \) (radiation) and \( A \) (absorption)
- Optically-dense limit (\( D \to 0 \))

Results obtained using a modified Roe-type solver with source upwinding.