Effect of sea roughness on bistatically scattered range coded signals from the Global Positioning System

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Abstract. A series of aircraft experiments was performed using a specialized GPS receiver and a nadir-oriented left hand circularly polarized antenna. This apparatus received reflections of the GPS signals from water surfaces under a variety of sea states. The cross-correlation between the reflected signal and a reference pseudo-random noise code was recorded as a function of the relative time delay. The shape of this function showed a dependence on the roughness of the reflecting surface. This dependence generally followed that predicted by theory for bistatic scattering of range-coded signals. Use of this information as a remote sensing technique for the determination of sea state is discussed.

Introduction

The existence of a strong reflected L-band signal from the Global Positioning System (GPS) off of calm water has been recognized by others [Parkinson, et al., 1996], [Auber, et al., 1974], [Garrison et al., 1997], usually as an interference to the normal applications of GPS for navigation. Recently, however, it has been proposed to use this signal for satellite or aircraft remote sensing [Katzberg and Garrison, 1996], [Neira, 1996], [Katzberg and Garrison, 1998], [Wu, et al., 1997]. One such application makes use of the effects of diffuse scattering from rough surfaces on the correlation properties of the pseudorandom noise (PRN) codes which form the basis of the GPS signal. In this application, the "widening" of the cross correlation between the received signal and the reference PRN sequence generated internal to the receiver is used as a measurement of the roughness of the sea surface [Katzberg, et al., 1998].

No previous experimental data was known to exist which could verify the effect on this cross-correlation predicted by theory. Therefore a series of aircraft flights was conducted using a specialized GPS receiver capable of recording this cross correlation function.

Theory

Except for a perfectly specular reflection, the reflected signal is composed of signals distributed over a range of relative time delays exceeding that from the specular point. The incoming GPS signal electric field can be represented in a simplified form as

\[ s_i(t) = A p(t) \]  

in which \( p(t) \) is the PRN sequence taking values of \{-1, +1\}. A diffusely reflected signal would be composed of an infinite sum of signals as a function of relative time delay, \( \delta \).

\[ s_{RD}(t) = \int_0^\infty a(\delta) p(t - t_s - \delta) d\delta \]  

In which, \( t_s \) is the additional time of travel from the specular point. The function \( a(\delta) \) is assumed to be a stationary random variable which gives the relative contribution to the electric field from reflecting points with delays of \( \delta \).

The geometry defining these time delays (expressed as path lengths by multiplication by the speed of light, \( c \)) is shown in Figure 1.

The power of the diffusely reflected signal described in equation (2) is the expected value \( P_{RD} = \langle s_{RD}^2(t) \rangle \). It is assumed that the correlation length (expressed in terms of the time delay) of the reflecting surface is short with respect to the size of one bit transition in the PRN code (referred to as a "code chip", 300 meters or 1 millisecond for the C/A code). This allows the approximation

\[ \langle a(\delta_1) a(\delta_2) \rangle \approx \begin{cases} \langle a(\delta)^2 \rangle, & |\delta_1 - \delta_2| < c t \rho \\frac{c}{c} \\ 0, & \text{otherwise} \end{cases} \]  

(3)

to be made, resulting in

\[ P_{RD} = \int_0^\infty \langle a^2(\delta) \rangle d\delta \]  

(4)

The term \( \langle a^2(\delta) \rangle \) can be interpreted as the power per unit of time delay at the relative delay of \( \delta \).

A GPS receiver, performs the cross correlation between the incoming signal and an identical replica of the PRN code for a given satellite. This correlation can be modeled as the integral

\[ R(\tau) = \int_{-\infty}^\infty s_{RD}(t) p(t - t_s - \tau) dt \]  

(5)

performed at some fixed delay \( \tau \) between the internally generated PRN code \( p(t - t_s - \tau) \) and the received signal. Equation (2) is substituted into (5), the square of the cross correlation is converted to a double integral and simplified.
Receiver

Locus of points with delay $c + \tau$

Figure 1. Scattering Geometry for Bistatically Reflected GPS Signals.

by the assumptions in equation (3). This results in the following

$$\langle R^2(\tau) \rangle = \int_0^\infty \langle a^2(\delta) \rangle A^2(\tau - \delta) d\delta$$

which states that the correlation output from the receiver is the convolution of the reflected signal power distribution with the function $A^2$. The $A$ function is the autocorrelation function of the PRN code, $A(\tau) \equiv \int p(t)p(t - \tau)dt$, and is given by

$$A(\tau) = \begin{cases} 1 - |\frac{\tau}{\tau_c}| & -\tau_c < \tau < \tau_c \\ 0, \text{otherwise} \end{cases}$$

This is an oversimplification of the process which occurs in a GPS receiver and the mechanisms of Doppler and phase difference removal [Parkinson, et al., 1996] have been ignored.

As the reflecting surface became rougher, the received power is distributed over a wider range of time delays and hence the function $R^2(\tau)$ has significant values over a wider range of time delays, $\tau$. To obtain quantitative results requires development of models which relate the structure of the reflecting surface to the distribution of power $a^2(\tau)$. A model for a surface with Gaussian distributed and exponentially correlated heights is derived in [Beckmann and Spizzichino, 1987] and applied to the problem of GPS signal reflections in [Katzberg and Garrison, 1996]. This model predicts the dependence of signal power on the ratio $\tan^2 \beta / \tan^2 \beta_0$. $\beta$ is a geometric angle defined as the angle between the surface normal and the plane formed by an incoming ray and a reflected ray as shown in Figure 1. $\beta_0$ is the ratio of the standard deviation of surface heights over their correlation length $(2\sigma/T)$. Larger values of $\beta_0$ mean higher average facet slopes which in turn means that at longer delays with respect to the specular point there is still significant probability that some facets will reflect in the receiver direction.

**Experiment Description**

A software configurable GPS receiver development system [GEC Plessey, 1996] was used as the basis for the experimental apparatus. A total of twelve (12) correlators were available with six (6) fed through the radio frequency (RF) front-end from a right hand circularly polarized antenna mounted on the roof of the aircraft fuselage. The other six correlators were fed through an identical circuit by a left hand circularly polarized antenna mounted on the bottom of the fuselage. The first six channels were used as a conventional GPS receiver which tracks up to six GPS satellites and generates a navigation solution. The bottom six channels, while synchronized by the same clock, operate entirely in an open-loop fashion.

A separate automatic gain control in both the upper and the lower RF front ends served to maintain a fixed value for the noise floor of the receiver. Each of the lower channel correlators was given the unique PRN and Doppler frequency of a corresponding top channel and are then sequentially stepped through 32 discrete time delays, at one per millisecond, separated by one half of a code chip. The sum square of the in-phase and quadrature components from these bottom correlators was recorded at each of these steps giving an experimental determination of the correlation power ($R^2(\tau)$) as a function of relative delay. All of these data were collected at the L1 frequency (1574.154 MHz) using the C/A code. Samples of the $R^2(\tau)$ function were passed through a discrete moving average filter and saved at a rate of once per second.

Data was collected on five different days between July and October 1997 on flights from the NASA Langley Research Center in Hampton, Virginia and on one flight in November 1997 out of the NASA Stennis Space Center in Mississippi.

A variation on this architecture was attempted in which the reference PRN code was generated faster than the received signal such that the relative delay ($\tau$) continuously varied at a fixed rate of change of 1 code chip every 2 milliseconds. The correlators were sampled once a millisecond, with the relative code difference reset after 32 milliseconds. This method, referred to as "sweeping", results in some small artifacts which are addressed in the results sections.

Data from the NOAA National Data Buoy Service [National Data Buoy Center, 1997] were monitored for sea state information.

Figure 2. Correlation Power as a Function of Relative Delay (Low Elevation Satellites).
Table 1. Summary of Aircraft Flights

<table>
<thead>
<tr>
<th>Satellite (SV)</th>
<th>7-28-97</th>
<th>10-20-97</th>
<th>10-22-97</th>
<th>11-7-97</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elev. (deg)</td>
<td>54</td>
<td>65</td>
<td>55</td>
<td>27</td>
</tr>
<tr>
<td>Az. (deg)</td>
<td>23</td>
<td>262</td>
<td>179</td>
<td>353</td>
</tr>
<tr>
<td>Altitude</td>
<td>3.9-4.9</td>
<td>5.4</td>
<td>6.7-7.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Range (km)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sig. Wave Hight (m)</td>
<td>0.72</td>
<td>2.47</td>
<td>1.11</td>
<td>0.59</td>
</tr>
<tr>
<td>Sea State</td>
<td>Slight</td>
<td>Rough to V. Rough</td>
<td>Moderate</td>
<td>Slight</td>
</tr>
<tr>
<td>Wind Spd. (m/s)</td>
<td>&lt; 1</td>
<td>12.3</td>
<td>8.8</td>
<td>7.7</td>
</tr>
<tr>
<td>Mean Wind Spd. (m/s)</td>
<td>2.9</td>
<td>12.1</td>
<td>9.3</td>
<td>8.5</td>
</tr>
<tr>
<td>Std. Dev. Wind Speed (m/s)</td>
<td>2.2</td>
<td>1.3</td>
<td>1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>( \beta_0 ) (deg)</td>
<td>6</td>
<td>14</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Beaufort No.</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>-5 dB Width</td>
<td>0.8</td>
<td>1.2</td>
<td>1.4</td>
<td>1.1</td>
</tr>
<tr>
<td>-10 dB Width</td>
<td>1.1</td>
<td>2.6</td>
<td>2.9</td>
<td>2.0</td>
</tr>
<tr>
<td>-15 dB Width</td>
<td>1.4</td>
<td>4.4</td>
<td>5.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

1Over 12 hour period.
2[Beckmann and Spitzichino, 1987], p 248.
3[Cox and Munk, 1954], p 68.

Data Reduction

In order to get a better approximation the the continuous shape of \( R^2(\tau) \), a cubic spline was fit through the average of the once-per-second samples. The peaks of each curve from the different measurement sets were then aligned and the curves normalized with respect to peak correlation power. This alignment was necessary because of the discrete half code chip sampling and the uncertainty in the absolute start of the delay bins. This allowed the determination of some quantitative measure of the effects of the "widening" of this function.

Results

The trend predicted by theory was that the correlation function will widen with increasing altitudes as well as with increasing sea roughness. However over altitude differences of a few kilometers the change with altitude was not pronounced. For this reason, the next section compares data from slightly different altitudes and shows the more significant dependence on sea state.

Figure 2 is a plot of these data, reduced as described in the previous section, for several low elevation (20 to 30 deg.) satellites indicating the widening of the correlation function for increasing wind speed. A similar plot for high elevation satellites is given in Figure 3. Wind speed is known to be related to the mean square slope of the surface [Cox and Munk, 1954].

These plots all demonstrate that most of the widening occurred in the direction of positive delay. This was a manifestation of the fact that the specular reflection represents the shortest path length and hence, no signal power could have be present at delays shorter than the specular point delay. No correlation power could have existed at delays earlier than 1 code chip before the specular reflection delay (because the \( A^2(\tau) \) is zero outside \( -1,1 \)).

Table 1 summarizes the satellite geometry, wind conditions, and wave height as represented by a number of different types of recorded data for each of the sets plotted in figures 2 and 3. The wind speed is given at the nearest hourly average to the time in which the GPS data was collected as well as an average over a 12 hour period. The latter was used because on one day (July 28) it was observed that...
there was a large fluctuation in wind speed and direction during the times in which the data was collected. Table 1 also lists the standard deviation of wind speed.

The widths of the correlation function at the -5dB, -10dB and -15dB points are given in Table 1 for comparison to the sea state.

A set of control data was obtained to identify any biases or artifacts which may be present in the receiver. Measurements were made of the correlation power for a direct signal. These showed some small deviation from the theoretical $\Delta^2(r)$, but very little power outside of the range of -1 to +1 code chip. The RF front end for the receiver was specified to have a bandwidth of 2 MHz.

A similar test was done for the “sweeping” receiver used on the July 1997 flight. Some widening was observed, but the total (two-sided) width of the correlation function was less than 2.5 code chips, as compared to the 2 code chip width predicted by theory.

The fundamental limitation on the accuracy and sensitivity on the design of delay-mapping receivers such as this is the signal to noise ratio in each of the delay bins. This can be improved by three different ways; increasing the integration time of the correlators; using a higher gain antenna; or using a lower noise preamplifier and front-end.

**Conclusion**

These early results demonstrate the trend of widening of the reported cross correlation function with roughness of the reflecting surface. This may have a future application as a measurement of sea state, similar to what is presently provided by active, monostatic, scatterometers. Theory has explained conceptually why this effect is manifested. It remains to reduce the resulting data to a useful measure of sea state. This will require the development of better models to describe the distribution of signal power as a function of relative delay as well as more organized flight campaigns aimed at correlating the findings with meaningful physical oceanographic quantities.

**References**


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